

SUBLIMATED DECOUPLING OF THE VORTEX LATTICE IN EXTREMELY TYPE-II LAYERED SUPERCONDUCTORS

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Abstract

The question of whether layer decoupling and vortex-lattice melting occur separately or not in the mixed phase of pristine layered superconductors in the extreme type-II limit is studied through a partial duality analysis of the layered XY model with uniform frustration. We find that both transitions occur simultaneously if the normal/superconducting transition of the vortex lattice in an isolated layer is first order and if a sufficient degree of layer anisotropy exists. We also find that a crossover to a highly entangled vortex lattice phase with relatively low phase rigidity across layers does not occur in practice under any circumstances at temperatures below the two-dimensional vortex-lattice melting transition.

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I. Introduction

It is now well established experimentally that the Abrikosov vortex lattice state in clean high-temperature superconductors undergoes a first-order melting transition into a liquid phase.¹ High-temperature superconductors are layered and extremely type-II.² The former vortex liquid phase in the most anisotropic materials like BSCCO is best described by a liquid of planar vortices inside of decoupled layers.³ A longstanding question is whether melting and layer decoupling occur simultaneously as a sublimation transition, or whether a separate decoupling transition follows the melting transition. Some experimental studies on the highly anisotropic BSCCO material show evidence for sublimation,^{4,5} while most experimental studies of the less anisotropic YBCO material point to separate melting and decoupling transitions.⁶

The experimental situation outlined above suggests that the degree of anisotropy is what in fact determines whether or not the vortex lattice in a layered superconductor sublimates. We shall study this proposal theoretically by analyzing the layered XY model with uniform frustration, which provides a qualitatively correct description of the thermodynamics deep inside of the mixed phase in extremely type-II layered superconductors.⁷ After performing a partial duality transformation on the XY model that is particularly well suited to the weak-coupling limit,⁸ we find that there can exist as many as three different decoupling transitions at temperatures $T_D < T_m < T_\times$, respectively. (We use the term ‘transition’ here loosely to describe both genuine phase transitions and cross-overs.) The phase correlation length across layers is equal to the inter-layer spacing along the dimensional crossover line^{9,10} at $T = T_\times$ that separates two-dimensional (2D) from three-dimensional (3D) vortex-liquid behavior.⁶ The phase correlation length across layers then either diverges or jumps to infinity along the melting line, $T = T_m$, which separates the superconducting and normal phases. Last, the crossover line $T = T_D$ that lies inside of the ordered phase is defined by the point at which the Josephson coupling energy reaches about half of its zero-temperature value. The macroscopic phase rigidity across layers becomes small in comparison to its zero-temperature value at temperatures $T > T_D$ because of the entanglement of fluxlines between adjacent layers.^{8,11} All three decoupling transi-

tions occur separately in the continuum regime at low perpendicular vortex density, but T_D crosses below the 2D melting temperature at only exponentially weak inter-layer coupling. At a moderate concentration of vortices, on the other hand, we find that the three decoupling transitions collapse onto a single sublimation line for weak enough Josephson coupling. This is due to the first-order nature of the melting of the 2D vortex lattice in such case. These results are compared with previous theoretical calculations based on the elastic medium description of the vortex lattice^{2,3,11,12} and with direct Monte Carlo simulation results of the XY model itself.⁷

II. Duality Theory

The layered XY model with uniform frustration is the minimum theoretical description of vortex matter in extremely type-II layered superconductors. Both fluctuations of the magnetic induction and of the magnitude of the superconducting order parameter are neglected within this approximation. The model hence is valid deep inside the interior of the mixed phase. The thermodynamics of the 3D XY model with anisotropy and uniform frustration is determined by the superfluid kinetic energy $E_{XY}^{(3)} = -\sum_{r,\mu} J_\mu \cos[\Delta_\mu \phi - A_\mu]|_r$, which is a functional of the superconducting phase $\phi(r)$ over the cubic lattice. Here, $J_x = J = J_y$ and $J_z = J/\gamma'^2$ are the local phase rigidities, with anisotropy parameter $\gamma' > 1$. The vector potential $A_\mu = (0, 2\pi f x/a, 0)$ represents the magnetic induction oriented perpendicular to the layers, $B_\perp = \Phi_0 f/a^2$. Here a denotes the square lattice constant, which is of order the zero-temperature coherence length, Φ_0 denotes the flux quantum, and f denotes the concentration of vortices per site. The component of the magnetic induction parallel to the layers is taken to be null throughout.

We shall now analyze the above layered system in the selective high-temperature limit, $k_B T \gg J_z$. Following ref. 8, the corresponding high-temperature expansion can be achieved through a *partial* duality transformation of the layered XY model along the z axis perpendicular to the layers. This leads to a useful layered Coulomb gas (CG) ensemble in terms of loops of Josephson vortices in between layers (fluxons).¹³ In particular, suppose that l denotes the layer index, that \vec{r} represents the x-y coordinates, and that $r = (\vec{r}, l)$. Phase correlations across N layers are then described by the phase auto-correlation function

probed at sites set by an integer field $p(r) = \delta_{\vec{r},0}(\delta_{l,1} - \delta_{l,N})$. These can be computed from the quotient

$$\left\langle \exp \left[i \sum_r p(r) \phi(r) \right] \right\rangle = Z_{\text{CG}}[p] / Z_{\text{CG}}[0] \quad (1)$$

of partition functions for a layered CG ensemble that describes the nature of the Josephson coupling:⁸

$$Z_{\text{CG}}[p] = \sum_{\{n_z(r)\}} y_0^{N[n_z]} \cdot \Pi_l C[q_l] \cdot e^{-i \sum_r n_z A_z}, \quad (2)$$

where $n_z(\vec{r}, l)$ is an integer field on links between adjacent layers l and $l+1$ located at 2D points \vec{r} . The ensemble is weighted by a product of phase auto-correlation functions

$$C[q_l] = \left\langle \exp \left[i \sum_{\vec{r}} q_l(\vec{r}) \phi(\vec{r}, l) \right] \right\rangle_{J_z=0} \quad (3)$$

for isolated layers l probed at the dual charge that accumulates onto that layer:

$$q_l(\vec{r}) = p(\vec{r}, l) + n_z(\vec{r}, l-1) - n_z(\vec{r}, l). \quad (4)$$

It is also weighted by a bare fugacity y_0 that is raised to the power $N[n_z]$ equal to the total number of dual charges, $n_z = \pm 1$. The fugacity is given by $y_0 = J_z / 2k_B T$ in the selective high-temperature regime, $J_z \ll k_B T$, reached at large model anisotropy. Also, the average number of n_z charges per link is equal to⁸ $2y_0(\langle \cos \phi_{l,l+1} \rangle - y_0)$, which is less than $J_z / k_B T$. This implies that the layered CG ensemble (2) is dilute in such case, because $y_0 \ll 1$. The former is required by the approximate nature of Eq. (2), which neglects multiple occupancy of the dual charges, n_z , on a given link. Last, the thermodynamics of the layered XY model is encoded by its partition function, which is given by the following product:

$$Z_{XY}^{(3)}[0] = [I_0(J_z / k_B T)]^{\mathcal{N}'} \cdot Z_{\text{CG}}[0] \cdot \Pi_l Z_{XY}^{(2)}[0]. \quad (5)$$

Here, $I_0(x)$ is a modified Bessel function, and $Z_{XY}^{(2)}[0]$ is the partition function of an isolated layer. Also, \mathcal{N}' denotes the total number of links between adjacent layers.

Interlayer correlations of the layered XY are easily determined using the CG ensemble (2) when the phase correlations within an isolated layer are short range.⁸ Let us introduce the notation $\phi_{l,l'}(\vec{r}) = \phi(\vec{r}, l') - \phi(\vec{r}, l)$ and take $A_z = 0$ due to the null magnetic field

parallel to the layers. A useful (in)equality for the autocorrelator between any number of layers, $n + 1$, can be computed to lowest order in the fugacity, y_0 . It reads⁸

$$\langle e^{i\phi_{l,l+n}} \rangle \leq \left[\mathcal{C}_0 \int \frac{d^2 q}{(2\pi)^2} \left(\frac{\mathcal{C}_q}{\mathcal{C}_0} \right)^{n+1} \right] (y_0 \mathcal{C}_0 / a^2)^n, \quad (6)$$

where

$$\mathcal{C}_q = \int d^2 r |C(\vec{r})| e^{i\vec{q} \cdot \vec{r}} \quad (7)$$

is the Fourier transform of the magnitude of the phase auto-correlation function (3) for an isolated layer probed at two points, \vec{r}_1 and \vec{r}_2 :

$$C(1, 2) = |C(\vec{r}_{12})| e^{-i \int_1^2 \vec{A}'(\vec{r}) \cdot d\vec{r}}, \quad (8)$$

where \vec{A}' is a suitably gauge-transformed vector potential (see below). Its magnitude depends only on the separation $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ between the probes, and it decays exponentially at separations beyond a characteristic correlation length ξ_{2D} due to the phase-incoherent state that is presently assumed. The layered CG ensemble (2) is therefore in a confining phase.¹⁴ The prefactor in brackets above in Eq. (6) typically decays polynomially with the separation n between layers. Also, Eq. (6) is an equality for $n = 1$,¹⁵ as well as for pure gauges such that $\vec{A}' = \vec{\nabla} \phi_0$ (see below). To conclude, the autocorrelator $\langle e^{i\phi_{l,l+n}} \rangle$ across layers decays at least exponentially with the separation n in the weak-coupling limit, $y_0 \rightarrow 0$, of the disordered phase.

The layered CG ensemble (2) can also be used to determine interlayer correlations in the ordered phase. Consider again an isolated layer, and suppose that general phase auto-correlation functions (3) are quasi-long range:

$$C[q] = g_0^{n_+} \cdot \exp \left[\eta_{2D} \sum_{(1,2)} q(1) \ln(r_{12}/r_0) q(2) \right] \cdot \exp \left(i \sum q \cdot \phi_0 \right), \quad (9)$$

where g_0 is equal to the phase rigidity of an isolated layer in units of J , where n_+ is equal to half the number probes, where r_0 is the natural ultraviolet scale of order the inter-vortex spacing, $a_{\text{vx}} = a/f^{1/2}$, and where $\phi_0(\vec{r})$ should resemble the unique zero-temperature configuration (independent of the layer index, l). The system of dual (n_z) charges in the

layered CG ensemble (2) is then in a plasma phase at low temperatures $\eta_{2D} < 2$.^{8,13} In such case, the macroscopic phase rigidity across layers is approximately given by⁸

$$\rho_s^\perp / J_z \cong \langle \cos \phi_{l,l+1} \rangle - y_0. \quad (10)$$

Furthermore, in this case an appropriate Hubbard-Stratonovich transformation of the CG partition function (2) in the absence of a source ($p = 0$) reveals that it is equal to the corresponding one $Z_{LD}[0] = \int \mathcal{D}\theta e^{-E_{LD}/k_B T}$ for a renormalized Lawrence-Doniach (LD) model up to a factor that is independent of the Josephson coupling, J_z . The corresponding energy functional is given by⁸

$$E_{LD} = \bar{J} \int d^2 r \left[\sum_l \frac{1}{2} (\vec{\nabla} \theta_l)^2 - \Lambda_0^{-2} \sum_l \cos(\theta_{l+1} - \theta_l) \right], \quad (11)$$

where $\bar{J} = k_B T / 2\pi \eta_{2D}$ is the macroscopic phase rigidity of an isolated layer,¹⁸ and where $\Lambda_0 = \gamma' a$ is the Josephson penetration length. The above continuum description (11) is understood to have an ultraviolet cut off of order the inter-vortex spacing, r_0 . A standard analysis of the product of partition functions (5) then yields that the strength of the local Josephson coupling is given by

$$\langle \cos \phi_{l,l+1} \rangle = y_0 + g_0 \langle \cos \theta_{l,l+1} \rangle, \quad (12)$$

where $\theta_{l,l+1} = \theta_{l+1} - \theta_l$.

To compute $\langle \cos \theta_{l,l+1} \rangle$ in the weak-coupling limit, it is sufficient to consider only layers l and $l+1$ in isolation from the rest of the system. At low temperature $\eta_{2D} \ll 1$, the harmonic approximation for the Josephson coupling term in Eq. (11) is valid: $\cos \theta_{l,l+1} \cong 1 - \frac{1}{2} \theta_{l,l+1}^2$. The resulting gaussian integration then yields $\langle \cos \theta_{l,l+1} \rangle = e^{-\langle \theta_{l,l+1}^2 \rangle / 2}$, with $\langle \theta_{l,l+1}^2 \rangle = \eta_{2D} \ln(\Lambda_J^2 / r_0^2)$. Here Λ_J is of order the Josephson penetration length, $\Lambda_0 = \gamma' a$. Substitution into Eq. (12) then produces the result⁸

$$\langle \cos \phi_{l,l+1} \rangle = y_0 + g_0 (r_0 / \Lambda_J)^{\eta_{2D}} \quad (13)$$

for the strength of the local Josephson coupling at low temperature $\eta_{2D} \ll 1$. The latter agrees with the result produced by analyzing a fermion analogy for the LD model (11),

as well as with an estimate by Glazman and Koshelev for the zero-field case ($r_0 \sim a$).¹⁰ Substitution of this result into Eq. (10) therefore yields the formula

$$\rho_s^\perp / J_z = g_0(r_0/\Lambda_J)^{\eta_{2D}} \quad (14)$$

for the macroscopic phase rigidity across layers in this regime.⁸ To conclude, macroscopic phase coherence exists across layers in the ordered phase (9).

III. Continuum Limit

We shall now review the phase diagram that results from employing the above duality analysis for the layered XY model in the continuum limit,⁸ $a \rightarrow 0$, which coincides with the regime of small perpendicular flux density, $f \ll 1/36$. In the absence of surface barriers, Monte Carlo simulations¹⁶ indicate that the vortex liquid phase of an isolated layer solidifies into a “floating” vortex lattice phase at the 2D melting temperature, $k_B T_m^{(2D)} \cong J/20$. A recent duality analysis of such a single layer finds that the standard 2D melting scenario¹⁷ takes place as long as rigid translations of the 2D vortex lattice are prohibited by surface barriers.¹⁹ In particular, general phase auto-correlation functions follow the form (9) in the vortex lattice phase at $T < T_m^{(2D)}$, with a 2D correlation exponent that takes on an *extremely* small value¹⁹ $\eta_{2D} \cong (28\pi)^{-1}$ just below the 2D melting temperature, $T_m^{(2D)}$. Further, η_{2D} decreases linearly to zero with decreasing temperature in the 2D vortex lattice. On the otherhand, the phase auto-correlation function (8) decays exponentially with separation in the *hexatic* phase that lies at temperatures just above $T_m^{(2D)}$. The associated correlation length, ξ_{2D} , diverges exponentially as temperature cools down to $T_m^{(2D)}$. The auto-correlation function retains, however, the trivial phase factor of the 2D vortex lattice:¹⁹ $\int_1^2 \vec{A}' \cdot d\vec{r} = \phi_0(2) - \phi_0(1)$.

We now illustrate that there exist as many as three distinct decoupling temperatures:⁸ $T_\times > T_m > T_D$. Consider the weak-coupling limit of the layered XY model, $\gamma' \rightarrow \infty$. Eq. (6) then becomes an equality in the hexatic phase of an isolated layer due to the trivial phase factor in the phase auto-correlation function (8).¹⁹ The phase correlation length across layers, ξ_\perp , is therefore equal to the spacing d between adjacent layers when

$$e^{-1} = y_0 \int d^2 r |C(\vec{r})|/a^2. \quad (15)$$

This defines a dimensional cross-over field,^{3,8–10}

$$f\gamma_{\times}'^2 \sim g_0(J/k_B T)(\xi_{2D}/a_{vx})^2 \quad (16)$$

in units of the naive decoupling scale Φ_0/Λ_0^2 , that separates 2D from 3D vortex-liquid behavior.⁶ It is traced out in Fig. 1. In these units, to be used hereafter, $f\gamma'^2$ gives the perpendicular field. The system is best described by a decoupled stack of 2D vortex liquids at fields above $f\gamma_{\times}'^2$. On the ordered side at $T < T_m^{(2D)}$, Eq. (14) for ρ_s^\perp implies that long-range order across layers exists: $\xi_\perp = \infty$. And since g_0J is equal to the phase rigidity of an isolated layer, Eq. (14) also implies that 3D scaling is violated at weak-coupling, $(r_0/\Lambda_J)^{\eta_{2D}} \ll 1$, in which case the phase rigidity across layers, ρ_s^\perp , is small in comparison to its value at zero temperature, J_z . This occurs at fields above the decoupling scale $f\gamma_D'^2 = e^{1/\eta_{2D}}$, however, which is astronomically large and of order 10^{38} at temperature below 2D melting due to the extremely small bound on the correlation exponent there,¹⁹ $\eta_{2D} < (28\pi)^{-1}$. At large anisotropy, $\gamma' > \gamma_D'$, the system is best described by an entangled stack of 2D vortex lattices¹¹ that exhibit a relatively small macroscopic Josephson effect.⁸ Last, the CG ensemble (2) indicates that a 3D vortex-lattice melting transition occurs at an intermediate temperature T_m when the typical distance between neighboring dual charges, $n_z = \pm 1$, grows to be of order ξ_{2D} , at which point these charges are confined into neutral pairs.^{8,14} It can be shown that T_m lies inside of the 2D-3D cross-over window $[T_m^{(2D)}, T_\times]$ by virtue of this definition (see ref. 8, Eq. 62). Also, by comparison with the layered CG ensemble (2) in zero field,^{8,19} the author has argued that in the weak-coupling limit, T_m marks the location of a second-order melting transition that separates the superconducting and normal phases. This means that $\xi_\perp(T)$ diverges as T cools down to T_m . A second-order transition in the vortex-liquid phase of YBCO that resembles the above has been reported recently.¹

Let us now determine what happens as interlayer coupling increases from the weak-coupling limit just studied. The n_z charges are screened at low temperature, $T < T_m^{(2D)}$, which means that no phase transition can take place as a function of the anisotropy parameter,⁸ γ' . Instead, a cross-over region exists for anisotropy parameters below γ_D' that separates a set of weakly coupled 2D vortex lattices at high field from a conventional

3D vortex lattice at low field. Again, the extremely small bound on the 2D correlation exponent η_{2D} at temperatures below 2D melting indicates that the former weakly coupled phase is not attainable there in practice. Eqs. (13) and (14) also imply that the Josephson effect is essentially independent of field/anisotropy at these temperatures, $T < T_m^{(2D)}$. This observation is consistent with Monte Carlo simulation results of the layered XY model with uniform frustration.⁷ On the disordered side, $T > T_m^{(2D)}$, the phase correlation length across layers, ξ_\perp , begins to grow larger than the spacing between adjacent layers at fields below $f\gamma_\times'^2$.⁸ Outside of the 2D critical region, at $\xi_{2D} \sim a_{vx}$, Monte Carlo simulations of the layered XY model with uniform frustration indicate that first-order melting occurs along the decoupling contour $\langle \cos \phi_{l,l+1} \rangle \sim 1/2$.^{7,15} The resulting phase diagram is depicted by Fig. 1.

IV. Sublimated Decoupling

We shall next apply the partial duality analysis outlined in section II to the layered XY model with only moderately small frustration. Let us consider again an isolated XY model over the square lattice, but with a uniform vorticity (frustration) between $1/30 < f < 1/2$. Monte Carlo simulations indicate that a depinning transition at $k_B T_p^{(2D)} = 1.5fJ$ now separates a pinned triangular vortex lattice at low-temperature from a vortex liquid phase at high temperature.¹⁶ The depinning transition is first order and no signs of a “floating” vortex-lattice phase are observed. Strict long-range phase correlations then exist at low temperatures $T < T_p^{(2D)}$ in the pinned phase, which implies that the phase auto-correlation functions are given asymptotically by Eq. (9) with $\eta_{2D} = 0$. Also, the disordered phase at high temperature $T > T_p^{(2D)}$ should be hexatic due to the underlying square-lattice grid.¹⁷ This means that the phase autocorrelations (8) exhibit exponential decay as well as a trivial phase factor:¹⁹ $\xi_{2D} < \infty$ and $\int_1^2 \vec{A}' \cdot d\vec{r} = \phi_0(2) - \phi_0(1)$. Below, we shall use these facts to map out the phase diagram of the layered XY model at such relatively high vortex density.

The first-order nature of the depinning transition in an isolated XY layer with relatively large uniform vorticity, $1/2 > f > 1/30$, implies that the phase correlation length is finite at temperatures just above the depinning transition: $\xi_{2D}(T_p^{(2D)}+) < \infty$. By Eq.

(16), the 2D-3D cross-over field here must also then be finite. Notice that $f\gamma'_\times$ is larger than unity at depinning if $\xi_{2D} > a_{vx}$ and if $g_0 \sim 1$, since $J > k_B T_p^{(2D)}$ for $f < 1/2$. Strict long-range phase coherence ($\eta_{2D} = 0$) exists on the low-temperature side at $T < T_p^{(2D)}$, however. We therefore reach the remarkable conclusion that at large anisotropy parameters of the corresponding layered XY model, $\gamma' \gg \gamma'_\times [T_p^{(2D)}]$, the line $T = T_p^{(2D)}$ marks a sublimation transition that separates a decoupled vortex liquid at $T > T_p^{(2D)}$ with essentially *no* interlayer phase coherence, $\xi_\perp < d$, from a pinned 3D vortex lattice state at $T < T_p^{(2D)}$ with long-range interlayer phase coherence, $\xi_\perp = \infty$. As depicted by Fig. 2, no 2D-3D cross-over regime exists in such case. Also, comparison of Eqs. (13) and (14) with the fact that the 2D correlation exponent η_{2D} vanishes in the low-temperature phase implies that the cross-over at $\gamma' = \gamma'_D(T)$ between weakly coupled and moderately coupled vortex lattices must collapse onto the depinning line at $T = T_p^{(2D)}$ and $\gamma' > \gamma'_\times [T_p^{(2D)}]$. Indeed, Eq. (13) indicates that the Josephson coupling $\langle \cos \phi_{l,l+1} \rangle$ is independent of field, $f\gamma'^2$, at temperatures below the sublimation transition and at such large anisotropy parameters. Last, the local Josephson coupling jumps down to a small value given by the vortex-liquid result,¹⁵ Eq. (6) at $n = 1$, once the vortex lattice sublimates. Similar jumps of order unity have been observed at vortex-lattice melting in BSCCO.⁵ In conclusion, the three possible decoupling transitions collapse onto a single sublimation transition! Such point-like as opposed to line-like melting of the vortex lattice has been observed in Monte Carlo simulations of the layered XY model with moderately small frustration.⁷

V. Discussion and Conclusions

Among the important theoretical results listed above is the local Josephson coupling in the vortex-lattice phase, Eq. (13), which can be expressed as $\langle \cos \phi_{l,l+1} \rangle = y_0 + g_0 e^{-\frac{1}{2}T/T_D(B_\perp)}$, with a temperature scale $k_B T_D(B_\perp) = 2\pi \bar{J} / \ln(B_\perp/B_\perp^*)$. Here, $B_\perp^* = \Phi_0/\Lambda_0^2$ is the naive decoupling field³ and $\bar{J} = k_B T / 2\pi \eta_{2D}$ is the 2D phase rigidity.¹⁸ As observed previously, the weak logarithmic field dependence above implies that $\langle \cos \phi_{l,l+1} \rangle$ is of order unity at low temperatures $T < T_m^{(2D)}$ and at perpendicular fields below the astronomically large scale $H_D \sim 10^{38} B_\perp^*$. The local Josephson coupling (13) shows essentially no field dependence in such case. This is confirmed directly by Monte Carlo simulations of

the layered XY with low uniform frustration.⁷ Despite the fact that the decoupled vortex-lattice state characterized by a small “cosine” does not exist in practice at temperatures below 2D melting, it is nevertheless remarkable that $T_D(B_\perp)$ coincides, to within a large numerical constant, with the temperature scale for layer decoupling induced by the unbinding of topological defects of the vortex lattice known as “quartets”. These consist of two opposing dislocation pairs in parallel inside of a given layer.^{2,11} Comparison with the present results then indicates that layer decoupling is indeed due to such a “quartet” unbinding mechanism, but that this occurs only for exponentially weak Josephson coupling at temperatures below 2D melting (cf. ref. 12). Glazman and Koshelev have also calculated the local Josephson coupling $\langle \cos \phi_{l,l+1} \rangle$ within the 3D elastic medium description for the vortex lattice,³ where they find a much stronger dependence $T'_D(B_\perp) \sim (B_\perp^*/B_\perp)^{1/2} T_m^{(2D)}$ for the decoupling temperature scale with field, on the other hand.² This discrepancy is due to the fact that the elastic-medium approximation represents a continuum theory. It therefore accounts only for long-wavelength fluctuations of the phase difference across layers. In the weak-coupling limit, the dominant contribution to the “cosine” is due to short-wavelength phase fluctuations between adjacent layers. These fluctuations are missed by the 3D elastic medium approximation, and we believe that this is why the Glazman-Koshelev result³ underestimates the size of the decoupling temperature scale at weak coupling.

In conclusion, a partial duality analysis of the layered XY model with uniform frustration finds that sublimated melting/decoupling of the 3D vortex lattice occurs if (i) the superconducting-normal transition of an isolated layer is first-order and if (ii) a sufficient degree of layer anisotropy exists. Condition (i) is guaranteed at strong substrate pinning,¹⁶ $1/2 > f > 1/30$. It has also been emphasized that no decoupled vortex-lattice state exists at temperatures below 2D ordering except for exponentially weak Josephson coupling between layers (see Figs. 1 and 2). This is notably consistent with complementary calculations that include interlayer magnetic coupling, but that turn off the Josephson coupling.¹² It must be mentioned, however, that the magnetic coupling between layers is weak in the extreme type-II regime studied here, and that this coupling can in fact be incorporated into the present duality analysis (2) of the vortex lattice in layered superconductors via an

effective “substrate potential” for isolated layers (see ref. 12). The additional substrate consists of an array of commensurate pins that mimics the magnetic effect of the vortex lattice in adjacent layers. It can therefore only increase phase coherence (3) inside of each 2D vortex lattice.²⁰ This means that the bound, $\eta_{2D} < (28\pi)^{-1}$, on the phase correlation exponent of the 2D vortex lattice continues to hold. Hence, within the “substrate potential” approximation for magnetic coupling,¹² the decoupling crossover to an entangled vortex lattice¹¹ with $\rho_s^\perp \ll J_z$ does not occur in practice at temperatures below 2D melting in the extreme type-II regime [see Eq. (14)]. We remind the reader that rigid translations of the vortex lattice are assumed throughout to be prohibited by surface barriers (see ref. 19).

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References

1. F. Bouquet, C. Marcenat, E. Steep, R. Calemczuk, W.K. Kwok, U. Welp, G.W. Crabtree, R.A. Fisher, N.E. Phillips and A. Schilling, *Nature* **411**, 448 (2001).
2. G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, and V.M. Vinokur, *Rev. Mod. Phys.* **66**, 1125 (1994).
3. L.I. Glazman and A.E. Koshelev, *Phys. Rev. B* **43**, 2835 (1991).
4. D.T. Fuchs, R.A. Doyle, E. Zeldov, D. Majer, W.S. Seow, R.J. Drost, T. Tamegai, S. Ooi, M. Konczykowski and P.H. Kes, *Phys. Rev. B* **55**, R6156 (1997);
5. T. Shibauchi, T. Nakano, M. Sato, T. Kisu, N. Kameda, N. Okuda, S. Ooi and T. Tamegai, *Phys. Rev. Lett.* **83**, 1010 (1999); M.B. Gaifullin, Y. Matsuda, N. Chikumoto, J. Shimoyama and K. Kishio, *Phys. Rev. Lett.* **84**, 2945 (2000).
6. E.F. Righi, S.A. Grigera, G. Nieva, D. Lopez and F. de la Cruz, *Phys. Rev. B* **55**, 14156 (1997); X.G. Qiu, V.V. Moshchalkov and J. Karpinski, *Phys. Rev. B* **62**, 4119 (2000).
7. A.E. Koshelev, *Phys. Rev. B* **56**, 11201 (1997).
8. J.P. Rodriguez, *Phys. Rev. B* **62**, 9117 (2000); *Physica C* **332**, 343 (2000); *Europhys. Lett.* **54**, 793 (2001).
9. S. Hikami and T. Tsuneto, *Prog. Theor. Phys.* **63**, 387 (1980); C. Kawabata, M. Takeuchi, S.R. Shenoy and A.R. Bishop, *J. Phys. Soc. Jpn.* **69**, 194 (2000).
10. L.I. Glazman and A.E. Koshelev, *Zh. Eksp. Teor. Fiz.* **97**, 1371 (1990) [*Sov. Phys. JETP* **70**, 774 (1990)].
11. M. Feigel'man, V.B. Geshkenbein, and A.I. Larkin, *Physica C* **167**, 177 (1990); E. Frey, D.R. Nelson, and D.S. Fisher, *Phys. Rev. B* **49**, 9723 (1994).
12. M.J.W. Dodgson, V.B. Geshkenbein and G. Blatter, *Phys. Rev. Lett.* **83**, 5358 (1999).
13. S.E. Korshunov, *Europhys. Lett.* **11**, 757 (1990).
14. A. Polyakov, *Phys. Lett.* **72 B**, 477 (1978).
15. A.E. Koshelev, *Phys. Rev. Lett.* **77**, 3901 (1996).
16. M. Franz and S. Teitel, *Phys. Rev. B* **51**, 6551 (1995); S.A. Hattel and J.M. Wheatley,

- Phys. Rev. B **51**, 11951 (1995).
17. D.R. Nelson and B.I. Halperin, Phys. Rev. B **19**, 2457 (1979).
 18. J.M. Kosterlitz, J. Phys. C **7**, 1046 (1974).
 19. J.P. Rodriguez, Phys. Rev. Lett. **87**, 207001 (2001).
 20. C.E. Creffield and J.P. Rodriguez, “Optimum Pinning of the Vortex Lattice in Extremely Type-II Layered Superconductors” (cond-mat/0205231).

Figure Captions

- Fig. 1. Shown is the proposed phase diagram for the layered XY model with uniform frustration in the continuum regime, $f \ll 1/36$. Notice the absence (in practice) of a decoupling transition at temperatures below 2D melting. Rigid translations of the vortex lattice are assumed to be prohibited by surface barriers. The mean-field temperature dependence $J \propto T_{c0} - T$ is also assumed.
- Fig. 2. The proposed phase diagram for the layered XY model with moderate uniform frustration, $1/30 < f < 1/2$ is displayed. The mean-field temperature dependence $J \propto T_{c0} - T$ is assumed once again.

**Erratum: “Sublimated decoupling of the vortex lattice
in extremely type-II layered superconductors”,
[Phys. Rev. B 66, 214506 (2002)]**

J.P. Rodriguez

The decoupling field for temperatures that lie below the 2D ordering transition that was derived in the discussion following Eq. (16) is more generally given by

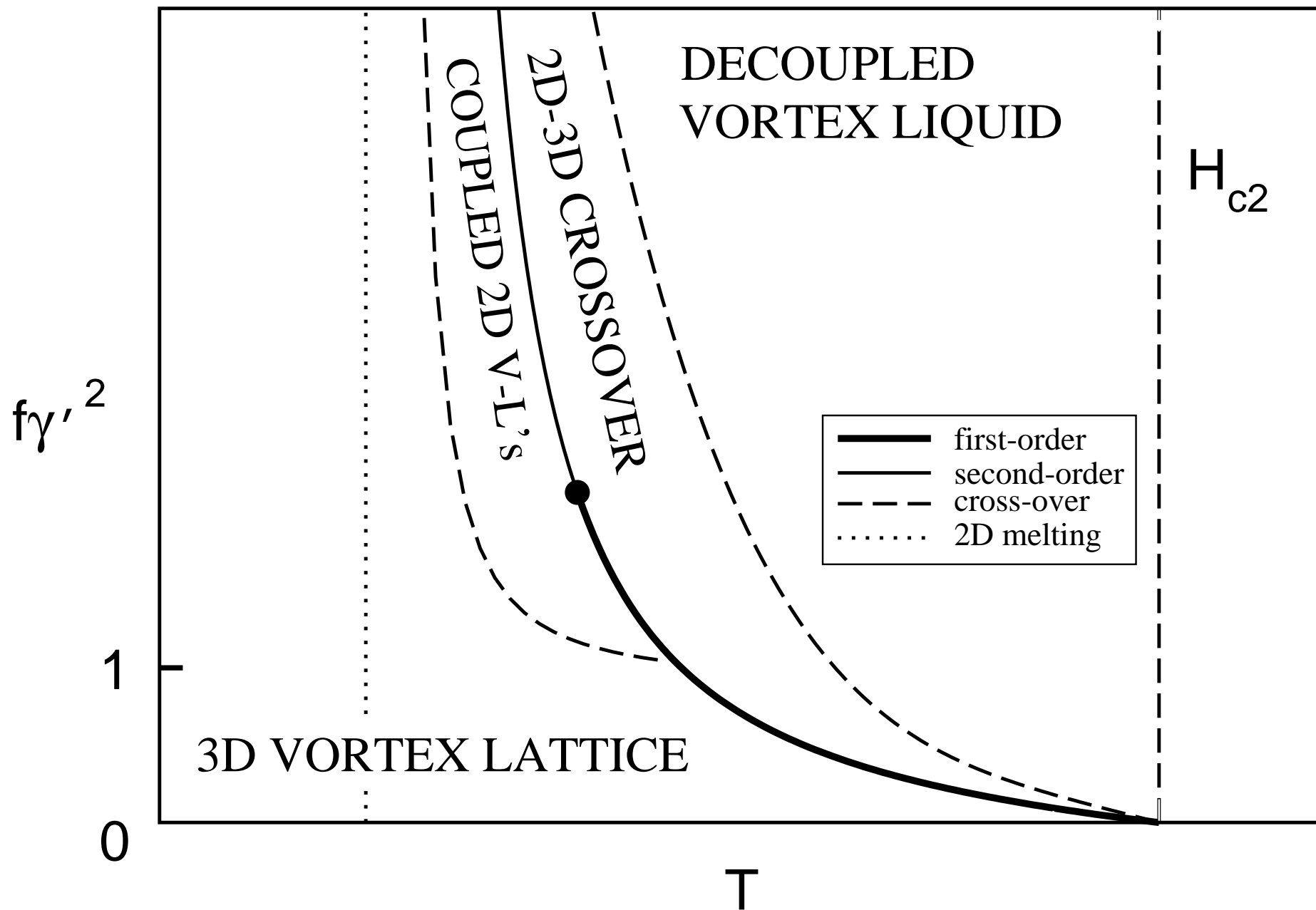
$$f\gamma_D'^2 = (r_0/a_{\text{vx}})^2 e^{1/\eta_{2D}},$$

where $r_0 \sim a_{\text{vx}}$ was implicitly assumed. Although the latter is not necessarily true, the ratio r_0/a_{vx} must be larger than $\kappa^{-1} = \xi_0/\lambda_0$. We have $\kappa \sim 100$ in YBCO for example. The above then implies that the decoupling field is bounded by $f\gamma_D'^2 > 10^{34}$ at temperatures below 2D ordering in such case, since $\eta_{2D} < (28\pi)^{-1}$. It therefore remains exponentially big.

More seriously, the claim made in section IV that the 2D phase correlation exponent is null at temperatures that lie below the 2D vortex-lattice depinning transition is incorrect. What is null is its vortex component, which leaves the spin-wave result $\eta_{2D} = k_B T / 2\pi J$ for the net exponent. The sentences in the middle of both paragraphs of section IV that begin with “Strict long-range phase ...” must therefore be replaced with “Quasi long-range phase ...”. Also, the equation “ $\eta_{2D} = 0$ ” that appears in both of these sentences must be replaced with “ $\eta_{2D} = k_B T / 2\pi J$ ”. The rest of section IV remains valid for Josephson coupling that is not exponentially weak. The equation displayed above, for example, yields an astronomically large lower bound $f\gamma_D'^2 > (r_0/a_{\text{vx}})^2 \cdot 10^{45}$ for the decoupling field at temperatures below 2D ordering and at an in-plane vortex concentration of $f = 1/25$. This bound is due to the value $k_B T_p^{(2D)} = 0.06J$ of the first-order transition temperature of an isolated layer in such case.

The above corrections do not change any of the conclusions drawn in the paper.

(NO BULK PINNING)



(SUBSTRATE PINNING)

